University of Bahrain

College of Information Technology
Department of Computer Science

ITCS253 Discrete Structures II
First Semester 2013/2014

Exam #2 – 60 Minutes

STUDENT NAME	
STUDENT#	
SECTION	

This exam contains 4 pages (including this cover page) and 6 questions. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You are allowed to use Calculators.

You *are not allowed* to use books, notes, or mobiles

Question	Points	Score
1	6	6
2	6	5.5
3	4	4
4	4	4
5	4	4
6	6	6
Total:	30	29.5

Instructor: Dr. Ali Alsaffar   Sections# 1 & 2

	identical no order	diff order
no rep	$c(n,r)$	$p(n,r)$
with many rep	$\binom{n+r-1}{r}$	n^r

(1) **[6 points]** How many 5-digit numbers we can have if

(a) Repeat is allowed and order matters. $n = 10$ $r = 5$

$(10)^5 = 100000$

(b) Repeat is not allowed and order matters.

$p(10,5) = \frac{10!}{(10-5)!} = \frac{10!}{5!} = 30240$

(c) Repeat is not allowed and order does not matter.

$\binom{10}{5} = \frac{10!}{(10-5)!5!} = 252$

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(d) Repeat is allowed and order does not matter.

$\binom{10+5-1}{5} = \binom{14}{5} = 2002$

(2) **[6 points]** In how many ways can you choose four countries to visit in four consecutive months from a list of nine possible countries.

no repition

order is important

9 countries → 4

(a) France must be visited?

4 8 7 6

$|F| = 8 \times 7 \times 6 \times 4 = 1344$

(b) France and Germany must be visited. $|F \cap G| = |F| + |G| - |F \cup G|$

$|F \cap G| = P(4,2) \times 7 \times 6 = 504$

$= 4 \times 3 \times P(7,2)$

$= P(4,2) \times P(7,2)$

(c) France or Germany must be visited. $|F \cup G| = |F| + |G| - |F \cap G|$

$|F \cup G| = 8 \times 7 \times 6 \times 4 + 8 \times 7 \times 6 \times 4 = 2688$

$4P(8,3) + 4P(8,3) - P(4,2) \times P(7,2)$

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(d) France or Germany must be visited but not both.

$|F \oplus G| = |F \cup G| - |F \cap G|$

$= 2688 - 504 = 2184$

$= 4P(8,3) + 4P(8,3) - 2P(4,2) \cdot P(7,2)$

(3) A class of students are taking an exam of five true/false questions.

(a) **[2 points]** How many different answers the students can have for the exam.

$(2)^5 = 32$ ✓

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(b) **[2 points]** If the class has 33 students, show that at least two students have the same answers?

$N = 33 \quad k = 32 \quad \lceil \frac{33}{32} \rceil = 2$ ✓

using the pigeonhole principle

(4) **[4 points]** Find the value of n in $2P(n, \overset{\text{constant}}{2}) + 50 = P(2n, 2)$. (n-2+1)

$2 \cdot \frac{n!}{(n-2)!} + 50 = \frac{(2n)!}{(2n-2)!}$

$2 \cdot \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} + 50 = \frac{2n(2n-1)\cancel{(2n-2)!}}{\cancel{(2n-2)!}}$

$2n(n-1) + 50 = 2n(2n-1)$

$2n^2 - 2n + 50 = 4n^2 - 2n$

$2n^2 - 4n^2 - \cancel{2n} + \cancel{2n} + 50 = 0$

$= -2n^2 + 50 = 0$ ✓

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$\sqrt{n^2} = \sqrt{\frac{-50}{-2}}$

$n = 5$ minuse is rejected

$\therefore$ The value of n is 5

straight forward

$2n(n-1) + 50 = (2n)(2n-1)$

(5) **[4 points]** For any nonnegative integers n and r with $r \le n$. Show that

$$\binom{n+1}{r} = \left[\frac{n+1}{r}\right]\binom{n}{r-1}$$

L.H.S $\frac{(n+1)!}{(n+1-r)!\,r!}$

R.H.S $\frac{n+1}{r} \cdot \frac{n!}{(n-r+1)!\,(r-1)!}$

$\frac{(n+1)\,n!}{r(r-1)!\,(n-r+1)!}$ ✓

$= \frac{(n+1)!}{r!\,(n+1-r)!} = \binom{n+1}{r} =$ L.H.S

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$= \frac{(n+1)n!}{r(n-r+1)(r-1)!} = \left[\frac{n+1}{r}\right]\binom{n}{r-1}$

(6) **[6 points]** Choose ONLY one of the following questions.

(a) In the expansion of $(3x^5 - \frac{1}{x^3})^7$ what is the coefficient of x^3?

$3x^5 - x^{-3}$

$$\sum_{k=0}^{7} \binom{7}{k} (3x^5)^{7-k} (-x^{-3})^k$$

$$\sum_{k=0}^{7} \binom{7}{k} (3)^{7-k} (-1)^k (x)^{35-5k} (x)^{-3k}$$

$$x^{35-5k} \cdot x^{-3k} = x^{35-5k-3k} = x^{35-7k}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

$$(3x^5 + (-x^{-3}))^7 = \sum_{k=0}^{7} \binom{7}{k} (3x^5)^{7-k} (-x^{-3})^k$$

$$= \sum_{k=0}^{7} \binom{7}{k} 3^{7-k} (-1)^k \cdot x^{35-5k} \cdot x^{-3k}$$

$$= \sum_{k=0}^{7} \binom{7}{k} 3^{7-k} \cdot (-1)^k x^{35-8k}$$

$\Rightarrow 35 - 8k = 3 \quad k = 4$

coefficient is

$\binom{7}{4} 3^3 (-1)^4$

(b) For an integer $n \geq 0$, prove that

$$3^n = 4^n - \binom{n}{1}4^{n-1} + \binom{n}{2}4^{n-2} - \binom{n}{3}4^{n-3} + \cdots + (-1)^n$$

$(4-1)^n = \sum_{k=0}^{n} \binom{n}{k} (4)^{n-k} (-1)^k$ using the binomial theorem

$$= \binom{n}{0}(4)^{n-0}(-1)^0 + \binom{n}{1} 4^{n-1} (-1)^1 + \binom{n}{2} 4^{n-2} (-1)^2 +$$

$$\binom{n}{3} 4^{n-3} (-1)^3 + \cdots + \binom{n}{n} 4^{n-n} (-1)^n$$

$$= 4^n - \binom{n}{1} 4^{n-1} + \binom{n}{2} 4^{n-2} - \binom{n}{3} 4^{n-3} + \cdots + (-1)^n$$

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